

Evaluation of Public Policies and Social Programs

Session 3: Traditional CBA II — Consumer Surplus & Market-Based Valuation

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Today

1. Foundations of consumer surplus, formalized
2. Application: continuous goods (highway toll)
3. Application: discrete goods (solar subsidy)
4. Deep discussion and what we learned

No paper presentation today — a roster change moved it to Session 7 (see the syllabus's Paper Presentations section).

Learning Objectives

By the end of today's session, you will be able to:

- Calculate consumer and producer surplus, and know precisely what welfare question each answers
- State the formal difference between compensating and equivalent variation (Hicks, 1943)
- Know when ordinary (Marshallian) consumer surplus is, and is not, a reliable approximation to the “true” Hicksian welfare measure (Willig, 1976)
- Apply welfare measures to a continuous-price change and to a discrete subsidy program

Foundations of Consumer Surplus

Recall: Harberger's first postulate

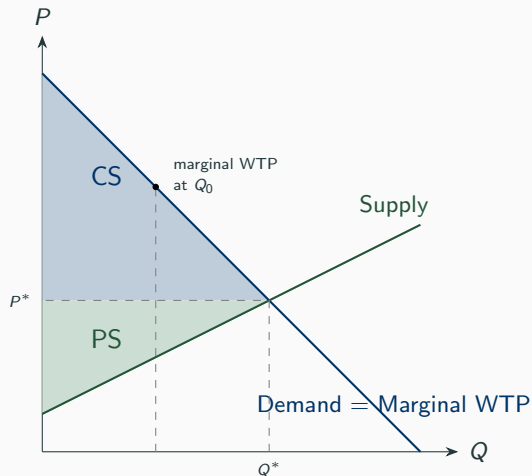
Session 1: the competitive market price of a good measures its value at the margin, and surplus changes read off observed demand and supply curves are valid welfare measures.

Today's question: *which* surplus measure, exactly — and how good an approximation is it?

WTP and consumer surplus

- Willingness to pay (WTP) and consumer surplus
- Marginal WTP vs. total WTP
- The demand curve *is* marginal WTP
- Area under the demand curve = total WTP

The four concepts, visually



Total WTP for Q^* units = the whole area under demand from 0 to Q^* = $CS + \text{Expenditure}$ ($P^* \times Q^*$). The curve's height at any Q is that unit's marginal WTP — here, at Q_0 .

Formalizing welfare change: Hicks (1943)

Hicks, J.R. (1943), “The Four Consumer’s Surpluses,” *Review of Economic Studies*

Before Hicks, “consumer surplus” was a single, ambiguous concept. Hicks showed there are (at least) two well-defined money-metric measures of a price change, which need not coincide:

Compensating Variation (CV)

The income change, *at the new prices*, that would bring the consumer back to their *original* utility

Equivalent Variation (EV)

The income change, *at the original prices*, that would bring the consumer to their *new* utility

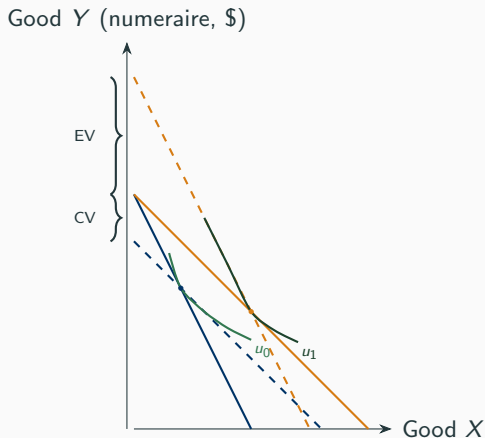
Why CV and EV can differ

Both are exact, theoretically well-defined welfare measures — but they price the same change using different reference prices, so they need not be equal when income effects are present.

$CV \approx EV$ when income effects are small.

But “small” compared to what? Hicks doesn’t tell us. Willig does.

The Hicksian diagram: CV and EV



CV, EV: vertical (income) gaps between actual and hypothetical “compensated” budget lines. Good Y is the numeraire ($p_Y = 1$), so a unit of Y costs \$1 and these vertical gaps are dollar amounts, not physical units of some other good. Standard Hicksian diagram, following Hicks (1943).

When can we just use ordinary consumer surplus? Willig (1976)

Willig, R.D. (1976), “Consumer’s Surplus Without Apology,” AER

The (Marshallian) area under the ordinary demand curve is not, in general, equal to CV or EV. But Willig derives a bound on the approximation error: it is small when the *income elasticity* of demand for the good and its *expenditure share* in the consumer’s budget are both small.

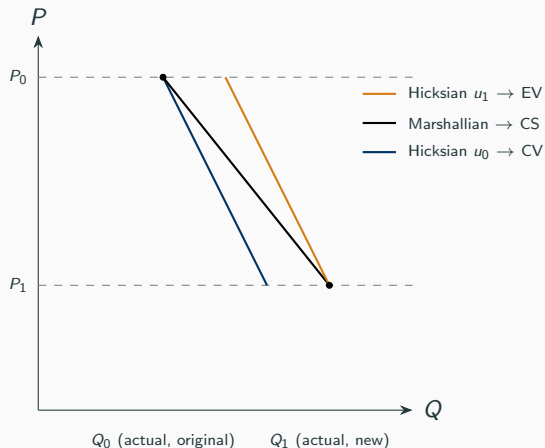
This is the formal justification for the applied convention: use ordinary CS unless the good is a large share of the budget or a large price change is involved.

Why does the gap open up? Substitution vs. income effects

- A price change moves demand through two channels: **substitution** (relative prices changed) and **income** (purchasing power changed)
- Marshallian CS uses the demand curve you actually *observe* — it bundles both channels together, with no way to tell them apart
- CV and EV instead hold utility fixed (Hicksian, “compensated” demand) — compensating the consumer’s income away precisely cancels the income channel, leaving only substitution

The CS/CV/EV gap is a pure measure of how much the income channel matters for this good — nothing else.

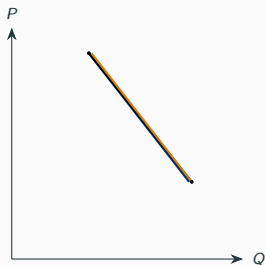
Three demand curves, one wedge



All three curves agree at the two *actually observed* bundles; they fan apart in between exactly to the extent the good has an income effect. Standard Marshallian/Hicksian demand comparison, following Hicks (1943) and Willig (1976).

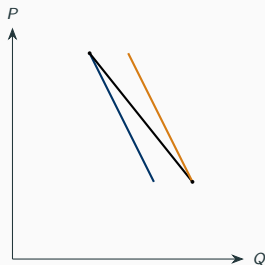
Same formula, two very different pictures

Toll: $\eta CS/(2M) \approx 0.01\%$



Curves nearly on top of each other —
 $CV \approx CS \approx EV$

Solar: $\eta CS/(2M) = 2.5\%$



Curves visibly fan apart — $CV < CS < EV$, a
real \$300K spread

Same three-curve logic, same formula — the only thing that changed between the two applications is how wide the fan is, because η and CS/M both got bigger.

Application 1: Continuous Goods

Highway toll reduction

Setup

Toll falls from \$5 to \$2. Demand: $Q = 10,000 - 1,000P$.

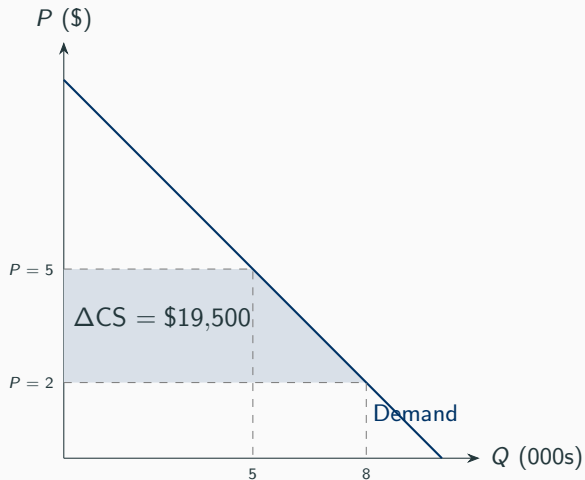
- Old: $P = \$5$, $Q = 5,000$
- New: $P = \$2$, $Q = 8,000$

$$\Delta CS = 0.5 \times (5,000 + 8,000) \times (\$5 - \$2) = \$19,500$$

$$\Delta PS = \$2 \times 8,000 - \$5 \times 5,000 = -\$9,000$$

$$\text{Net benefit} = \$10,500$$

Highway toll: the ΔCS trapezoid



From CS to CV and EV: applying Willig's bound

Willig's approximation (income elasticity η , aggregate income M of affected commuters):

$$CV \approx CS \left(1 - \frac{\eta}{2} \cdot \frac{CS}{M} \right), \quad EV \approx CS \left(1 + \frac{\eta}{2} \cdot \frac{CS}{M} \right)$$

Illustrative parameters

Income elasticity of demand for highway travel $\eta = 0.5$ (note: *income* elasticity, not price elasticity); aggregate annual income of the corridor's commuters $M \approx \$50\text{M}$.

Solution: CV and EV

$$\frac{\eta}{2} \cdot \frac{CS}{M} = \frac{0.5}{2} \times \frac{19,500}{50,000,000} \approx 0.0000975 \text{ (0.01\%)}$$

$$CV \approx 19,500 \times (1 - 0.0000975) \approx \$19,498$$

$$EV \approx 19,500 \times (1 + 0.0000975) \approx \$19,502$$

CV, CS, and EV differ by about \$4 out of \$19,500 — practically identical.

This is Willig's formula *confirming* the qualitative claim from the last slide with an actual number, not just asserting it.

Deep discussion: is ΔCS good enough here?

- Toll spending is very likely a small share of a commuter's income — Willig's conditions plausibly hold, and the calculation on the previous slide confirms it numerically
- So ordinary $\Delta CS = \$19,500$ is an excellent stand-in for CV and EV *here*
- But: is this true for *every* commuter, or just on average? The calculation used one aggregate income figure — what happens to $\eta \cdot CS/M$ if we redo it for low-income commuters specifically, with a much smaller M ? (A question we'll return to with distributional weights, Session 8.)

Application 2: Discrete Goods

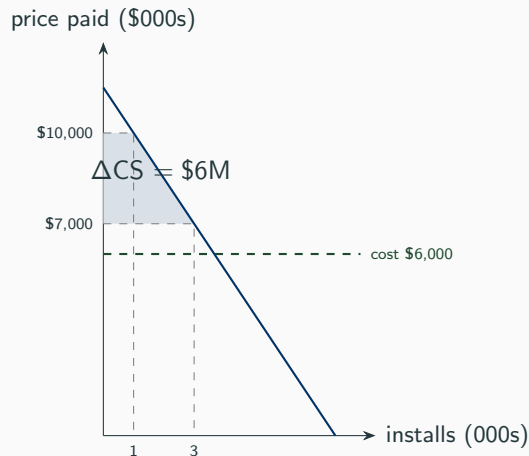
Setup

- Subsidy: \$3000 per installation
- Without subsidy: 1000 installations/yr at \$10,000 each
- With subsidy: 3000 installations/yr at \$7,000 each
- Installation cost (supply): constant at \$6,000

Task (pairs, 12 minutes)

1. Draw supply and demand
2. Calculate the change in consumer surplus
3. Calculate government cost
4. Calculate the change in installer (producer) surplus
5. Is there deadweight loss or deadweight gain? Why?

Solar subsidy: the demand-side picture



The demand curve alone gives ΔCS ; the flat cost line is a separate reference for the producer-surplus and government-cost parts of the task.

From CS to CV and EV: a bigger gap this time

Same formula as the toll example, deliberately more demanding parameters:

Illustrative parameters

Income elasticity of demand for rooftop solar $\eta = 2$ (a discretionary, big-ticket durable good — much higher than highway travel's 0.5); aggregate household income of the 3,000 installing households, at \$80,000/household, $M \approx \$240\text{M}$.

Solution: CV and EV

$$\frac{\eta}{2} \cdot \frac{CS}{M} = \frac{2}{2} \times \frac{6,000,000}{240,000,000} = 0.025 \text{ (2.5\%)}$$

$$CV \approx 6,000,000 \times (1 - 0.025) = \$5,850,000$$

$$EV \approx 6,000,000 \times (1 + 0.025) = \$6,150,000$$

	Value	vs. ordinary CS
CV	\$5.85M	−2.5%
CS (ordinary)	\$6.00M	—
EV	\$6.15M	+2.5%

A \$300,000 spread — 250× wider (in percentage terms) than the toll example. Same formula, and the answer changes qualitatively.

Deep discussion: does Willig's approximation still hold?

A \$10,000 $\rightarrow$ \$7,000 price fall is a 30% change on a good that is a large share of most households' annual budget.

- Willig's bound gets *worse* exactly here: large price change, plausibly large expenditure share, and a much higher income elasticity — and the previous slide shows this isn't just a qualitative worry, it's a real \$300K (2.5%) gap
- Would you still trust ordinary $\Delta CS = \$6M$ as a stand-in for CV/EV in this application, given the calculation? Why or why not?
- What would a more careful (Hicksian) analysis require that ordinary CS does not?

Wrap-Up

Most of what matters for the Astoreca project has no market price.

School quality. Student achievement.

Preview: Session 6 — Revealed Preference.

What We Learned Today

- Hicks formalized welfare measurement into (at least) two distinct, well-defined concepts — CV and EV — that need not coincide
- Willig tells us precisely *when* we can skip that complexity and use ordinary consumer surplus: small income elasticity, small expenditure share
- The size of the price change and the good's budget share are not just technical details — they determine whether our welfare numbers are trustworthy
- Non-market goods (Sessions 6–7) will need an entirely different strategy, because there is no demand curve to read a price off of in the first place

- Problem Set 1 (self-study checkpoint: by Session 7)
- Re-read the Fundación Astoreca case packet ahead of the guest session